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Active Magnetic Damping Attitude Control for Gravity Gradient Stabilized Spacecraft

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The problem of the general formulation of a magnetic auxiliary control strategy for a gravity gradient stabilized spacecraft is presented. Magnetic torque is orthogonal to the external magnetic field; therefore, it is impossible to generate precisely the control torque that would be required according to simple control theories, such as the proportional derivative, model reference, etc. However, it is possible to define the desirable torque first and then derive from this the actual control torque. Two different criteria are presented for this derivation and a class of control algorithms is then formulated, based on the model reference approach. Stability is discussed in general and specifically for the algorithms. Finally, a comparison among the different proposed strategies and with respect to another approach known from literature has been performed numerically.

Nomenclature

B	= Earth magnetic induction magnitude
$\boldsymbol{B}$	= Earth magnetic induction field
c_{θ_j}	= $\cos\theta_j$
E	= Lyapunov candidate function
E_{ry}	= system energy of roll–yaw motion
e	= orbit eccentricity
$\boldsymbol{G}$	= spacecraft gravity gradient torque
i	= coil current
$\boldsymbol{J}$	= spacecraft inertia
$\boldsymbol{K}_d$	= derivative gain matrix
$\boldsymbol{K}_p$	= proportional gain matrix
k_d	= derivative gain
k_p	= proportional gain
$\boldsymbol{M}$	= coil dipole moment
m	= nondimensionalized time
N	= number of coils
$\boldsymbol{Q}$	= kinematic matrix
$\tilde{\boldsymbol{R}}$	= local vertical unit vector
S	= coil section surface
s_{θ_j}	= $\sin\theta_j$
$\boldsymbol{T}$	= spacecraft torque with respect to the center of mass
$\boldsymbol{T}_c$	= spacecraft control torque
$\boldsymbol{T}_{ci}$	= ideal control torque
$\boldsymbol{T}_d$	= spacecraft disturbance torque
$\boldsymbol{x}$	= attitude vector
$\Delta\boldsymbol{T}_c$	= undesired control torque
θ_1	= roll Euler angle
θ_2	= pitch Euler angle
θ_3	= yaw Euler angle
μ	= Earth gravitational constant
$\boldsymbol{\omega}$	= spacecraft relative angular velocity
$\boldsymbol{\omega}_0$	= orbital angular velocity
$\boldsymbol{\omega}_t$	= spacecraft inertial angular velocity

Subscripts

1	= roll axis component
2	= pitch axis component
3	= yaw axis component

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Introduction

THE gravity gradient stabilization and control of spacecraft has been considered a very attractive method since the beginning of the space era due to its intrinsic simplicity and reliability. However, the main problem faced to get asymptotic stability is the damping of the oscillations. The purely passive stabilization has been proved to be rather unsatisfactory; hence, the attention focused on the active damping control. Nowadays, the gravity gradient stabilization procedure has re-emerged for the following applications: 1) control of very large spacecraft or space stations where the gravity gradient torque plays a major role with respect to other environmental torques, 2) the tethered satellite systems where gravity gradient is one of the main parameters governing the system dynamics, and 3) control of small scientific satellites because of the simplicity, reliability, and low cost of the gravity stabilization procedure.

The third application is the subject of this work. In this case asymptotic stability and low cost are achievable only if the active control is simple and economic, such as with active magnetic control. This kind of control has been studied, for instance, in Refs. 1–3, where different active magnetic control strategies have been developed for gravity gradient stable spacecraft.

The present paper is based on the fact that a systematic approach to general solution through the magnetic control is still lacking, and the procedures proposed in the literature are amenable to possible improvement. Therefore, the following items are developed: 1) general magnetic control algorithms, 2) a class of new algorithms based on a reference model technique, and 3) comparison among the proposed algorithms and that of Ref. 1.

Equations of Attitude Motion

The active damping control systems proposed are based on three orthogonal coils along the principal body axes. It is assumed that the spacecraft is rigid and the determination of the control currents is the main focus of this investigation. The attitude dynamics of the spacecraft is given by the Euler equations and the kinematic equations associated with the attitude parameters used, which, in this work, are the 2-1-3 sequence Euler angles, with θ_2 pitch, θ_1 roll, and θ_3 yaw, with respect to an ideal Earth-pointing frame.[4–6]

The Euler equations of motion around the center of mass of the spacecraft with constant inertia, are given by

$$\boldsymbol{J}\cdot\dot{\boldsymbol{\omega}} - \boldsymbol{J}\cdot(\boldsymbol{\omega}\times\boldsymbol{\omega}_0) + \boldsymbol{\omega}_t\times\boldsymbol{J}\cdot\boldsymbol{\omega}_t + \boldsymbol{J}\cdot\frac{\mathrm{d}\boldsymbol{\omega}_0}{\mathrm{d}t} = \boldsymbol{T} \tag{1}$$

with

$$\boldsymbol{\omega}_t = \boldsymbol{\omega} + \boldsymbol{\omega}_0 \tag{2}$$

where $(\dot{\ })$ is the time derivative in the body frame (BF); $\mathrm{d}(\ )/\mathrm{d}t$ is the time derivative in the inertial frame (IF); $\boldsymbol{\omega}_0$ is the orbital angular velocity vector; $\boldsymbol{\omega}_t$ is the spacecraft's inertial angular velocity vector, i.e., the velocity of BF with respect to IF; $\boldsymbol{\omega}$ is the spacecraft's relative angular velocity vector, i.e., the velocity of BF with respect to the stabilized frame; $\boldsymbol{T}$ is the external torque; and J is the inertia tensor.

The external torque $\boldsymbol{T}$ can be expressed as the sum of the following:

$$\boldsymbol{T} = \boldsymbol{T}_c + \boldsymbol{T}_d + \boldsymbol{G} \tag{3}$$

with

$$\boldsymbol{G} = (3\mu/R^3)\hat{\boldsymbol{R}} \times \boldsymbol{J} \cdot \hat{\boldsymbol{R}} \tag{4}$$

where $\hat{\boldsymbol{R}}$ is the unit local vector directed from the spacecraft to the Earth center.

By introducing the Euler angles θ_1, θ_2, and θ_3 and defining $\boldsymbol{x}^T = \{\theta_1, \theta_2, \theta_3\}$, the kinematic equations can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{Q} \cdot \boldsymbol{\omega} \tag{5}$$

with $\boldsymbol{Q}$ defined as follows:

$$\boldsymbol{Q} = \begin{bmatrix} c_{\theta_3} & -s_{\theta_3} & 0 \\ s_{\theta_3}/c_{\theta_1} & c_{\theta_3}/c_{\theta_1} & 0 \\ s_{\theta_3}s_{\theta_1}/c_{\theta_1} & c_{\theta_3}s_{\theta_1}/c_{\theta_1} & 1 \end{bmatrix}$$

Defining $\boldsymbol{r}$ as

$$\boldsymbol{r} = -\boldsymbol{J} \cdot (\omega \times \omega_0) + \boldsymbol{\omega}_t \times \boldsymbol{J} \cdot \boldsymbol{\omega}_t + \boldsymbol{J} \cdot \frac{\mathrm{d}\omega_0}{\mathrm{d}t} \tag{6}$$

Eq. (1) becomes

$$\boldsymbol{J} \cdot \dot{\omega} = \boldsymbol{T} - \boldsymbol{r} \tag{7}$$

Deriving Eq. (5) and substituting into Eq. (7) yields

$$\ddot{\boldsymbol{x}} = \dot{\boldsymbol{Q}} \cdot \omega + \boldsymbol{Q} \cdot \boldsymbol{J}^{-1} \cdot (\boldsymbol{T} - \boldsymbol{r}) \tag{8}$$

Equation (8) is a convenient formula for simulation purposes. To design the system it may be conceptually useful to refer to a linear model. Substituting Eq. (4) into Eq. (3), Eq. (3) into Eq. (1), and linearizing (see Ref. 4 for details), the spacecraft's equations of motion become

$$\begin{aligned} \theta_1'' + 4k_1\theta_1 + (1 - k_1)\theta_3' &= \tau_{c1} + \tau_{d1} \\ \theta_2'' + 3k_2\theta_2 &= 2e \sin m + \tau_{c2} + \tau_{d2} \\ \theta_3'' + k_3\theta_3 - (1 - k_3)\theta_1' &= \tau_{c3} + \tau_{d3} \end{aligned} \tag{9}$$

where I_1, I_2, and I_3 are the principal moments of inertia with respect to roll, pitch, and yaw; τ_{c1}, τ_{c2}, and τ_{c3} are nondimensional control torques; e is assumed small; and

$$k_1 = \frac{I_2 - I_3}{I_1} \qquad k_2 = \frac{I_1 - I_3}{I_2} \qquad k_3 = \frac{I_2 - I_1}{I_3} \tag{10}$$

The derivatives are taken with respect to the nondimensionalized time $m = \omega_0 t$.

Magnetic Control Torque

Magnetic control is achieved by three mutually orthogonal coils whose dipoles are along the 1, 2, and 3 axes of the body. The resulting torque is

$$\boldsymbol{T}_c = \boldsymbol{M} \times \boldsymbol{B} \tag{11}$$

where $\boldsymbol{B}$ is the Earth's magnetic induction.[7]

Equation (11) states that $\boldsymbol{T}_c$ must be orthogonal to $\boldsymbol{B}$, which is a function of the orbital position of the spacecraft. Therefore, only control torques normal to $\boldsymbol{B}$ are possible. As a consequence, the generated control torque $\boldsymbol{T}_c$ is generally different from that desired, $\boldsymbol{T}_{ci}$, namely,

$$\boldsymbol{T}_c = \boldsymbol{T}_{ci} - \Delta\boldsymbol{T}_c \tag{12}$$

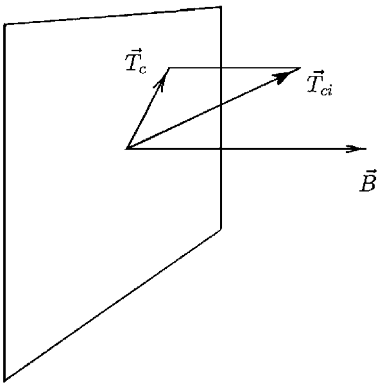

Fig. 1 Minimum control torque for a given *B*.

where $\Delta\boldsymbol{T}_c$ is an undesirable component of the control torque.

Two solutions are proposed.

1) Minimize the Euclidean norm of $\Delta\boldsymbol{T}_c$, i.e., $\|\boldsymbol{T}_c - \boldsymbol{T}_{ci}\| = \min$, which gives

$$\Delta\boldsymbol{T}_c = (\boldsymbol{T}_{ci} \cdot \hat{B})\hat{B} \tag{13}$$

where $\hat{B} = \boldsymbol{B}/B$, with $B = \|\boldsymbol{B}\|$. It is possible to demonstrate Eq. (13) through a simple geometric construction (Fig. 1). In fact, given vector $\boldsymbol{B}$, all $\boldsymbol{T}_c$ satisfying Eq. (11) lie in a plane orthogonal to $\boldsymbol{B}$. If $\boldsymbol{T}_{ci}$ is defined, the $\boldsymbol{T}_c$ vector that has the minimum distance from $\boldsymbol{T}_{ci}$ is obtained by projecting $\boldsymbol{T}_{ci}$ to the plane orthogonal to $\boldsymbol{B}$; hence, Eq. (13) is obtained.

2) Set two components of $\boldsymbol{T}_c$ equal to the corresponding components of $\boldsymbol{T}_{ci}$ in the body axes and obtain the third from the orthogonality between $\boldsymbol{T}_c$ and $\boldsymbol{B}$, stated by Eq. (11). Two options are proposed in this case: a) assign components $T_{c1} = T_{ci1}$ and $T_{c2} = T_{ci2}$, and determine T_{c3} from Eq. (11) or b) assign components $T_{c3} = T_{ci3}$ and $T_{c2} = T_{ci2}$, and determine T_{c1} from Eq. (11).

Once $\boldsymbol{T}_{ci}$ has been selected and $\boldsymbol{T}_c$ has been computed by using approach 1 or 2, it is necessary to compute the dipole moment $\boldsymbol{M}$ that achieves $\boldsymbol{T}_c$. The minimum $\|\boldsymbol{M}\|$ among the solutions of Eq. (11) is such that $\boldsymbol{M} \cdot \boldsymbol{B} = 0$. Using this option it is possible to demonstrate, from Eq. (11), that

$$\boldsymbol{M} = (\boldsymbol{B} \times \boldsymbol{T}_c)/B^2 \tag{14}$$

The coil driving current i is given by the equation

$$i = M/NS \tag{15}$$

where N is the number of turns and S is the coil section.

The design of a control algorithm is based, first, on the choice of the ideal torque $\boldsymbol{T}_{ci}$ and, then, on the generation of the actual torque $\boldsymbol{T}_c$ by means of approach 1 or 2. The $\boldsymbol{T}_{ci}$ can be variously defined according to different criteria (for instance, proportional derivative in Ref. 1, angular momentum in Ref. 2). The $\boldsymbol{T}_{ci}$ specifically referred to is assigned with a reference model technique. The subsequent step, which generates the actual control torque $\boldsymbol{T}_c$, may or may not present some major characteristics of $\boldsymbol{T}_{ci}$, such as stability. When stability is not automatically guaranteed, positive enforcing actions may be necessary, as will be discussed subsequently. A consequence of the considerations made to date is that a variety of different control methods may be generated and that an evaluation of their relative convenience becomes very desirable. In the "Results" section a numerical comparison among a set of possible algorithms will be presented.

Stability

From the linear theory of torque free gravity gradient stabilized spacecraft in circular orbits,[6] it is well known that there are four possible stable equilibrium configurations in the Lagrange zone, all having the minimum inertia axis aligned along the local vertical and the maximum along the orbit normal. When a control torque is applied, a practical problem is to locate the region of the spacecraft's initial attitudes that guarantees the asymptotic stability. The Lyapunov theorem states that an equilibrium solution $\boldsymbol{y} = 0$ is asymptotically stable if it is possible to find a function $E(\boldsymbol{y}, t)$, known as a Lyapunov function,[8,9] that satisfies the following requirements in a neighborhood η of $\boldsymbol{y} = 0$:

$$E(0,t) = 0 \qquad E(\boldsymbol{y},t) > 0 \qquad \forall t > 0$$
$$\frac{\mathrm{d}E}{\mathrm{d}t} < 0 \qquad \forall t > 0 \tag{16}$$

The η domain is the stability domain, and $\boldsymbol{y}$ is the state vector.

A natural choice for the Lyapunov candidate function is the mechanical energy of the free system. In fact, if it is assumed that the applied control torque is dissipative, this energy is continually decreasing and, due to its additive properties, can always be set to zero at equilibrium. The only condition that remains to be satisfied is its positive definiteness.

The system's mechanical energy of the linearized motion is given by

$$E = I_1\dot{\theta}_1^2 + I_2\dot{\theta}_2^2 + I_3\dot{\theta}_3^2 + 4\omega_0^2(I_2 - I_3)\theta_1^2 + 3\omega_0^2(I_1 - I_3)\theta_2^2 + \omega_0^2(I_2 - I_1)\theta_3^2 \tag{17}$$

which is positive definite when the system is Lagrange stable ($I_2 > I_1 > I_3$). Therefore, it is possible to conclude that Lagrange stable configurations, in the presence of dissipative torques, are stable at least in an infinitesimal neighborhood of the equilibrium position.

In the nonlinear motion it is possible to try an extension of function (17) by introducing the nonlinear expressions of the Euler angles and rates. It yields

$$\begin{aligned}E &= I_1\left(\dot{\theta}_2 c_{\theta_1} s_{\theta_3} + \dot{\theta}_1 c_{\theta_3}\right)^2 + I_2\left(\dot{\theta}_2 c_{\theta_1} c_{\theta_3} - \dot{\theta}_1 s_{\theta_3}\right)^2\\ &+ I_3\left(\dot{\theta}_3 - \dot{\theta}_2 c_{\theta_1}\right)^2 + (I_2 - I_3)\omega_0^2\left(4 - 3s_{\theta_2}\right)s_{\theta_1}^2\\ &+ (I_1 - I_3)3\omega_0^2 s_{\theta_2}^2 + (I_2 - I_1)\omega_0^2\left[\left(1 + 3s_{\theta_2}^2 - 4s_{\theta_1}^2 + 3s_{\theta_2}^2 s_{\theta_1}^2\right)s_{\theta_3}^2 + \tfrac{3}{2}s_{\theta_1}s_{2\theta_2}s_{2\theta_3}\right]\end{aligned} \tag{18}$$

If the configuration is purely Lagrange stable ($I_2 > I_1 > I_3$), a finite stability domain is generally to be expected. A sufficient stability condition independent of the initial conditions, however, can be obtained by upper bounding all negative terms and lower bounding all positive terms of the last three contributions of Eq. (18). This yields

$$\frac{I_2 - I_3}{I_2 - I_1} > \frac{9}{2} \tag{19}$$

Condition (19) is usually well satisfied for many Lagrange stable spacecraft. Unfortunately, a purely dissipative control torque hypothesis cannot always be met in practice. In fact, a restoring term, at least on the yaw angle, is often needed because the gravity gradient torque on this axis may be too low to ensure convenient response. Whenever the torque is not dissipative per se, it is possible to impose $\mathrm{d}E/\mathrm{d}t \le 0$ by switching off the control in those positions of the orbit where the control torque would produce an energy increase. This technique, on the one hand, undermines the control authority and on the other, avoids instability.

Linear Reference Model Control Method

The idea is to use the theory of the reference model linearizing feedback[5] to define an ideal control torque $\boldsymbol{T}_{ci}$ that is able to guarantee the desired behavior of the system. By referring first to the linearized equation (9), it is possible to see that the torque

$$\boldsymbol{T}_{ci} = \lambda\boldsymbol{x} + \nu\dot{\boldsymbol{x}}$$

with

$$\lambda = -\mathrm{diag}\{k_{p1}, k_{p2}, k_{p3}\} \qquad k_{pj} = \text{assigned proportional gains}$$

$$\nu = -\begin{bmatrix} k_{d1} & 0 & k_1 - 1\\ 0 & k_{d2} & 0\\ 1 - k_3 & 0 & k_{d3}\end{bmatrix}$$

$$k_{dj} = \text{assigned derivative gains}$$

decouples the equations by reducing the system to three damped oscillators. This defines the ideal torque; to obtain the real control torque, either approach 1 or 2 defined earlier may be followed.

Approach 2 is considered first. In approach 2 roll and pitch can be assigned,

$$\tau_{c1} = (1-k_1)\theta_3' - k_{p1}\theta_1 - k_{d1}\theta_1' \qquad \tau_{c2} = -k_{p2}\theta_2 - k_{d2}\theta_2' \tag{20}$$

or yaw and pitch can be assigned,

$$\tau_{c3} = -(1-k_3)\theta_1' - k_{p3}\theta_3 - k_{d3}\theta_3' \qquad \tau_{c2} = -k_{p2}\theta_2 - k_{d2}\theta_2' \tag{21}$$

where the proportional and derivative gains k_{pj} and k_{dj} are assumed to be nonnegative. The third component of the control torque is a consequence of the orthogonality condition given in Eq. (11).

Substituting expressions (20) and (21) in the linearized equations (9), the system becomes

$$\begin{aligned}&\theta_1'' + k_{d1}\theta_1' + [k_{p1} + 4k_1]\theta_1 = 0\\ &\theta_2'' + k_{d2}\theta_2' + [k_{p2} + 3k_2]\theta_2 = 0\\ &\theta_3'' + (B_1/B_3)(I_1/I_3)(1 - k_1)\theta_3' + k_3\theta_3 = (B_1/B_3)(I_1/I_3)\\ &\quad\times(k_{p1}\theta_1 + k_{d1}\theta_1') + (B_2/B_3)(I_2/I_3)(k_{p2}\theta_2 + k_{d2}\theta_2')\end{aligned} \tag{22}$$

$$\begin{aligned}&\theta_1'' - (B_3/B_1)(I_3/I_1)(1 - k_3)\theta_1' + 4k_1\theta_1 = (B_3/B_1)(I_3/I_1)\\ &\quad\times(k_{p3}\theta_3 + k_{d3}\theta_3') + (B_2/B_1)(I_2/I_1)(k_{p2}\theta_2 + k_{d2}\theta_2')\\ &\theta_2'' + k_{d2}\theta_2' + [k_{p2} + 3k_2]\theta_2 = 0\\ &\theta_3'' + k_{d3}\theta_3' + [k_{p3} + k_3]\theta_3 = 0\end{aligned} \tag{23}$$

Note that in both systems the damping coefficient of the yaw equation depends on the sign of the magnetic field component ratio B_1/B_3 (B_1 = the roll component and B_3 = the yaw component of the magnetic induction vector in body coordinates); in particular, the damping is positive for $B_1/B_3 > 0$ in Eq. (22) and for $B_1/B_3 < 0$ in Eq. (23). Therefore, a combined control approach of Eqs. (22) and (23) could be assumed, based on switching according to the sign of the B_1/B_3 ratio, that is, actuate the roll and pitch coils according to Eq. (20) when B_1/B_3 is positive, or according to Eq. (21) when B_1/B_3 is negative.

The two systems (22) and (23), when considered individually, appear uncoupled and stable for $k_{pj} > 0$ and $k_{dj} > 0$. When switched, however, θ_1 and θ_3 are bilaterally coupled, because θ_3 depends on θ_1 in Eq. (22) and θ_1 depends on θ_3 in Eq. (23). The θ_2 equation, on the contrary, maintains the same form in both Eqs. (22) and (23), and this ensures its stability under $k_{p2} > 0$ and $k_{d2} > 0$. Moreover, the energy associated with the pitch motion is nonincreasing. The coupled θ_1–θ_3 system responds to the excitation due to the pitch motion, but this excitation does not influence its stability characteristics. Therefore, the stability of θ_1–θ_3 may be assessed by considering, as the Lyapunov candidate function, the reduced roll–yaw energy E_{ry}, and the time variations caused on this function by the θ_1–θ_3 dependent part of the right-hand term of Eqs. (22) and (23).

Namely, the roll–yaw energy can be defined for system (22) as

$$E_{\mathrm{ry}} = I_1\theta_1'^2 + I_3\theta_3'^2 + I_1(4k_1 + k_{p1})\theta_1^2 + I_3k_3\theta_3^2$$

A similar expression for system (21) is

$$E_{\mathrm{ry}} = I_1\theta_1'^2 + I_3\theta_3'^2 + I_14k_1\theta_1^2 + I_3(k_3 + k_{p3})\theta_3^2$$

The variations of these energies due to the θ_1–θ_3 dependent excitation terms are, respectively,

$$\frac{1}{2}\frac{\mathrm{d}E_{\mathrm{ry}}}{\mathrm{d}m} = -I_1\left[k_{d1}\theta_1'^2 + \frac{B_1}{B_3}(1 - k_1)\theta_3'^2 - \frac{B_1}{B_3}(k_{d1}\theta_1' + k_{p1}\theta_1)\theta_3'\right] \tag{24}$$

and

$$\frac{1}{2}\frac{\mathrm{d}E_{\mathrm{ry}}}{\mathrm{d}m} = I_3\left[-k_{d3}\theta_3'^2 + \frac{B_3}{B_1}(1-k_3)\theta_1'^2 + \frac{B_3}{B_1}(k_{d3}\theta_3' + k_{p3}\theta_3)\theta_1'\right] \tag{25}$$

A sufficient stability condition is easily derived when the reference gain is purely derivative, i.e., $k_{p1} = k_{p3} = 0$. In this case the derivative of the roll–yaw energy is negative definite when

$$k_{d1} < 4(B_3/B_1)(1-k_1) \qquad k_{d3} > 4(B_1/B_3)(1-k_3)$$

A variant of the method is obtained by defining variable gain coefficients as follows:

$$\begin{aligned} K_{d1} &= k_{d1}|B_3/B_1| & K_{d2} &= k_{d2}|B_3/B_2| \\ K_{p1} &= k_{p1}|B_3/B_1| & K_{p2} &= k_{p2}|B_3/B_2| \end{aligned} \tag{26}$$

when $B_1/B_3 > 0$ and

$$\begin{aligned} K_{d3} &= k_{d3}|B_1/B_3| & K_{d2} &= k_{d2}|B_1/B_2| \\ K_{p3} &= k_{p3}|B_1/B_3| & K_{p2} &= k_{p2}|B_1/B_2| \end{aligned} \tag{27}$$

when $B_1/B_3 < 0$.

In this case the stability conditions are independent of the magnetic field ratios and the following constraints shall be met:

$$k_{d1} < 4(1-k_1) \qquad \text{and} \qquad k_{d2} > 4(1-k_3) \tag{28}$$

Of course, stability can also be obtained outside the given conditions, by applying explicit energy control, i.e., switching off the coils when $\mathrm{d}E/\mathrm{d}t > 0$.

Now approach 1 is considered. The linearized equations become

$$\begin{aligned} \theta_1'' + k_{d1}\theta_1' + [k_{p1} + 4k_1]\theta_1 &= -\Delta T_{c1} \\ \theta_2'' + k_{d2}\theta_2' + [k_{p2} + 3k_2]\theta_2 &= -\Delta T_{c2} \\ \theta_3'' + k_{d3}\theta_3' + [k_{p3} + k_3]\theta_3 &= -\Delta T_{c3} \end{aligned} \tag{29}$$

where it is evident that equations are coupled through the ΔT_{cj} terms. In this instance general stability is not automatically possible, and so only switching off, under the $\mathrm{d}E/\mathrm{d}t > 0$ condition, results in stable solutions.

Although these considerations, based on the linearized system, are important for understanding and designing the system, only simulations based on fully nonlinear dynamics of the actual system provide the actual behavior of the controlled spacecraft.

In the nonlinear case it can easily be seen, from Eq. (1), that the ideal torque

$$\begin{aligned} \boldsymbol{T}_{ci} = (\boldsymbol{J}\cdot\boldsymbol{Q}^{-1}\cdot\boldsymbol{J}^{-1})\cdot\big[&(\boldsymbol{J}\cdot\boldsymbol{Q}\cdot\boldsymbol{J}^{-1})\cdot\boldsymbol{r} \\ &-(\boldsymbol{J}\cdot\dot{\boldsymbol{Q}})\cdot\boldsymbol{\omega} - \boldsymbol{K}_D\cdot\dot{\boldsymbol{x}} - \boldsymbol{K}_P\cdot\boldsymbol{x}\big] \end{aligned} \tag{30}$$

reduces the dynamics to the following reference behavior:

$$\boldsymbol{J}\cdot\ddot{\boldsymbol{x}} + \boldsymbol{K}_D\cdot\dot{\boldsymbol{x}} + \boldsymbol{K}_P\cdot\boldsymbol{x} = \boldsymbol{0} \tag{31}$$

with

$$\boldsymbol{x} = \{\theta_1, \theta_2, \theta_3\}^T \qquad \boldsymbol{J} = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$$

$$\boldsymbol{K}_D = \begin{bmatrix} k_{d1} & 0 & 0 \\ 0 & k_{d2} & 0 \\ 0 & 0 & k_{d3} \end{bmatrix}$$

$$\boldsymbol{K}_P = \begin{bmatrix} k_{p1} + 4\omega_0^2(I_2 - I_3) & 0 & 0 \\ 0 & k_{p3} + 3\omega_0^2(I_1 - I_3) & 0 \\ 0 & 0 & k_{p3} + \omega_0(I_2 - I_1) \end{bmatrix}$$

Therefore, three decoupled damped oscillators are again ideally obtained by applying $\boldsymbol{T}_{ci}$. From this torque the real control torque can be computed by using approach 1 or 2 and switching off the coils when the mechanical energy of the system increases.

Results

Equations (1–5), with T_{ci} specified by Eq. (30) and T_c computed from approach 1 or 2, have been used for computer simulations. The gravity gradient torque and the magnetic field model given in Ref. 4 have also been implemented, including a Keplerian orbit model and aerodynamic disturbance torque. Three variants of the linearizing feedback model reference method have been used for computer simulations: MR (model reference) DT (residual delta torque), approach 1; MR AB (switching), approach 2, constant gains and alternate switching for roll and yaw [Eqs. (20) and (21)]; and MR AB VG (variable gains), MR AB with gains variable as function of magnetic field ratios [Eqs. (26) and (27)].

All these methods have been compared with that of Ref. 1 (named CO for Cornell University) method and have been numerically investigated for almost equal conditions regarding power consumption, identical initial conditions, and no disturbances. Figures 2–5 show the attitude and electric power consumption for the MR-AB approach. Only derivative gains have been considered for these simulations in accordance with the choice made, based on the stability considerations described. In Tables 1–6, the performances and parameters used for the methods proposed are listed in further detail.

Another element of comparison is behavior given the external disturbances. For this purpose a standard disturbance has been defined (Table 4). The system responses under these disturbances and for the other conditions (identical to the described test cases) are shown in Figs. 6–9, and the results summarized in Table 6.

The system performances are compared referring to 1) the peak angles during the transient phase, 2) the characteristic time to reach a steady state (e.g., below 0.1-deg attitude error), and 3) the asymptotic angles under disturbances.

Tables 5 and 6 clearly show that the MR-AB method gives the best performances, avoiding high-peak angles and showing faster convergence. The asymptotic response, on the contrary, is the same for all cases.

Table 1 Spacecraft inertia and start conditions

Moments of inertia, kg/m^2	Initial angles, deg	Initial Euler rates, rad/s
$I_1 = 338$	10	0
$I_2 = 353$	10	0
$I_3 = 66$	10	0

Table 2 Orbital parameters

Argument of perigee, deg	Inclination, deg	Initial anomaly, deg	Eccentricity	Altitude, km
0	0	0	0	300

Table 3 Actuator parameters

Number of coils	Applied tension, V	Cross section diameter, cm
212	28	220

Table 4 Aerodynamic parameters

Center of pressure-center of mass distance 1, 2, 3, m			Length, m	Diameter, m	Atmospheric density at 300 km, kg/m^3
0	0	−0.02	2.4	1.2	2.75×10^{-11}

Table 5 Test case with no disturbance

Test case	Method	Derivative gains $kd1$	k_{d2}	k_{d3}	Number of orbits for steady state	Electric energy consumption j	Peak angles, deg Roll	Pitch	Yaw
1a	CO	70,000	100,000	20,000	25	3,200	10	10	44
2a	MR DT	20	40	0.1	22	3,300	10	10	44
3a	MR AB	0.9	0.5	0.3	17	3,200	10	10	23
4a	MR AB VG	1.0	7.5	0.3	25	3,300	10	10	25

Table 6 Test case with aerodynamic disturbance

Test case	Method	Derivative gains k_{d1}	k_{d2}	k_{d3}	Number of orbits to reach steady state	Peak angles, deg Roll	Pitch	Yaw
1b	CO	70,000	100,000	20,000	27	10	10	41
2b	MR DT	20	40	0.1	25	10	10	30
3b	MR AB	0.9	0.5	0.3	20	10	10	22
4b	MR AB VG	1.0	7.5	0.3	30	10	10	25

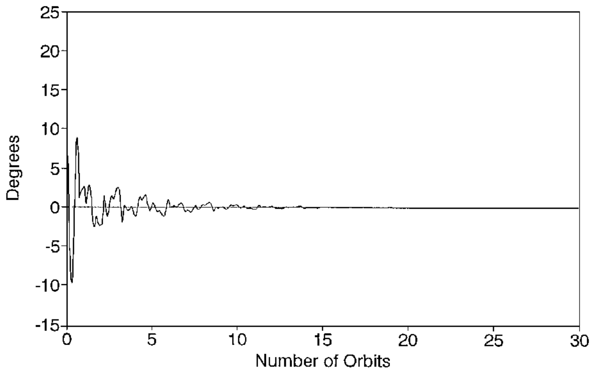

Fig. 2 Test case 3a: roll angle.

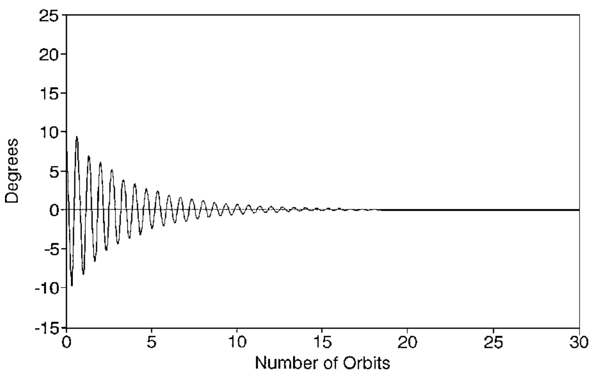

Fig. 3 Test case 3a: pitch angle.

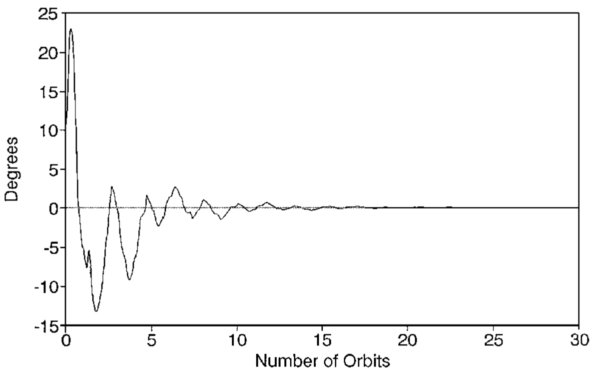

Fig. 4 Test case 3a: yaw angle.

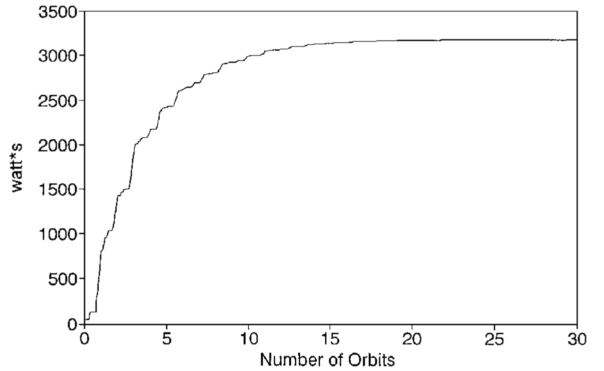

Fig. 5 Test case 3a: consumed energy.

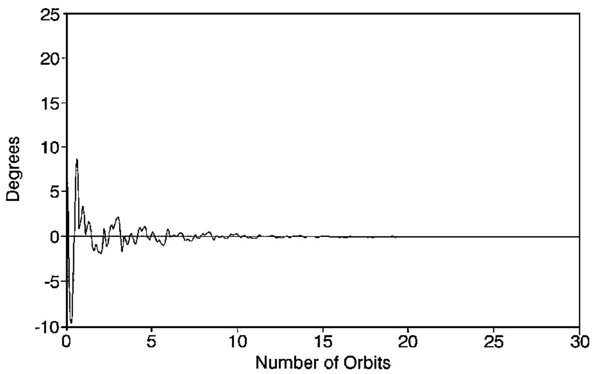

Fig. 6 Test case 3b: roll angle.

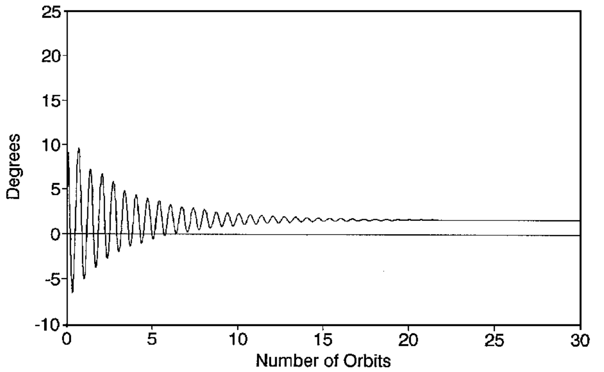

Fig. 7 Test case 3b: pitch angle.

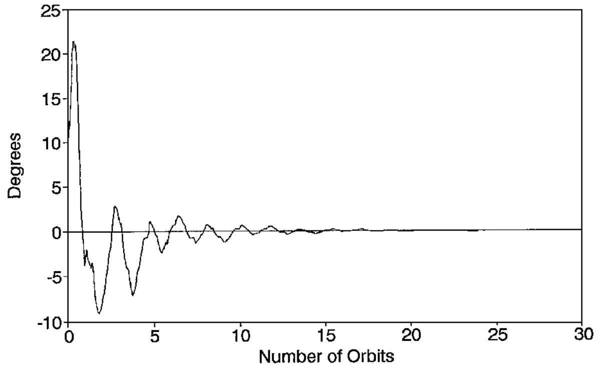

Fig. 8 Test case 3b: yaw angle.

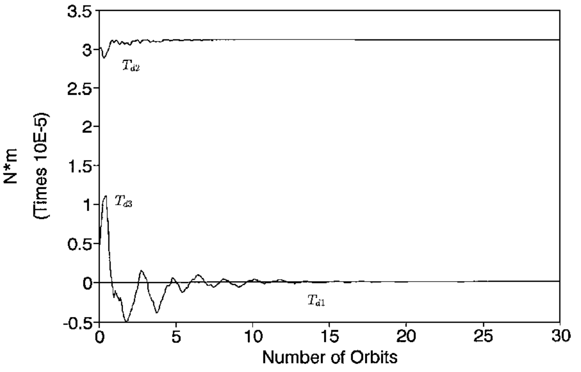

Fig. 9 Test case 3b: aerodynamic torque.

Conclusions

A linear reference model magnetic damping control strategy for a gravity gradient stabilized spacecraft has been developed. With the reference model control theory an ideal control torque has been generated, tending to force a reference behavior corresponding to decoupled damped oscillators in pitch, roll, and yaw. The magnetic torque, however, must be orthogonal to the magnetic induction vector at each point of the orbit and, therefore, cannot generate exactly the desired action. Two approaches have been proposed for generating the actual control torque beginning with the ideal torque. The first method is based on the torque closest to the ideally sought, the second on torques with two components equal to the ideal. According to these methods three different model-reference-based algorithms have been formulated. Stability has been discussed proposing, in general, the free system mechanical energy as a Lyapunov candidate function and an unconditional stability criterion for dissipative control torques. Asymptotic stability is, therefore, always obtainable by monitoring in-flight the system energy and switching the magnetotorquers off whenever the energy tends to increase. A priori criteria of reference model gains selection have been indicated for two of the proposed approaches, which can consequently also obtain stability without positive energy control. Finally, the proposed strategies have been compared with each other (and with respect to the approach of Ref. 1) by means of numerical simulations in various representative conditions and with mostly identical power consumption. The results show the actual stable operation of the proposed algorithms and the better performance of the one indicated as MR AB with respect to all others, including that of Ref. 1.

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